On the Non-Linear Distortion in the Reproduction of Phonograph Records Caused by Angular Deviation of the Pickup Needle

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(Incl. 5 illustrations)

1. Introduction

The recording of gramophone records with the usual laterally cut groove is made, as is generally known, by means of a cutting head which is advanced parallel to itself along a radius of the lacquer with constant speed, while the cutting stylus performs small oscillations in a radial direction. These oscillations produce small lateral displacements of the spiral groove. These lateral displacements form a curve which, in principle, is analogous to the particle movement of the acoustic wave. Perfectly faithful reproduction of the record can only be achieved if the tracking of the groove by the needle of the sound box or the electrical phonograph pick-up takes place under the same geometrical conditions as the recording. In reality, most of the time this is only approximately the case. Almost all of the purely acoustic gramophones use the well known device with pivoted tone arm for guiding the sound box across the record, and also the electrical playback apparatus are usually built in a similar way. With this commonly used apparatus the needle on the one hand describes an arc, hence not a radial path, and on the other hand the sound box or phonograph pick-up rotates with respect to the tangent of the groove. Many other guiding mechanisms for electrical phonograph pick-ups are suggested in order to maintain a continuously tangential relationship, but so far these have not been implemented with the simplicity, convenience and freedom from resonances that are usually achieved with pivoted tone arms. The question now is whether a most favourable configuration can be found for the pivoted tone arm which minimises the deviation from the ideal case to such an extent that in practice noticeable drawbacks do not occur.

This important problem in gramophone technology has been known for a long time and has been dealt with in the literature [1-6]) from various points of view, without a clear solution being proposed. As early as in 1924 P. Wilson [1]) published an analysis of the problem, and proposed formulae for a configuration which is optimal under certain conditions. Later, the author [3]) performed an investigation under similar conditions, which led to somewhat simpler configuration rules. In recent years, interest in this question has increased, particularly since electrical phonograph pick-ups have been designed which can satisfy the highest requirements for the quality of reproduction [6,7]).

In the following, the problem is developed further by calculating the resulting non-linear distortion, and, taking this distortion into consideration, by discussing the most appropriate configuration.

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[1]) P. Wilson, Needle-track alignment. The Gramophone, Sept. 1924, p. 129-131

[2]) P. Wilson and G.W. Webb, Modern gramophones and electrical reproducers, London 1929, p. 124-132

[3]) E. Löfgren, Nålföringen vid grammofoner. Radio (Sweden) 7 (1929), no. 19, p. 15-18 and 24, no. 22-23, p. 10-14

[4]) E.A. Chamberlain, Correct pick-up alignment. Wirel. Wld. 26 (1930), no. 13, p. 339-340

[5]) F. Record, Gramophone tracking. J. Sci. Instrum. 9 (1932), no. 9, p. 286-289

[6]) J.R. Bird and C.M. Chorpening, The offset-head crystal pick-up. Radio Engng. 17 (1937), no. 3, p. 16-18

[7]) F.V. Hunt and J.A. Pierce, A radical departure in phonograph pick-up design. Electronics 11 (1938), no. 3, p. 9-12

2. General considerations

If the tracking of a record with a laterally cut groove is to be called theoretically correct, the plane through the needle tip and the longitudinal axis of the needle (needle plane) must be perpendicular to the record and tangential to the groove at the needle tip. Additionally, there is of course the condition that the needle tip does not experience substantial shift in the groove direction, a condition that is satisfactorily achieved in all practical cases. As a simple calculation shows it is in this regard without noticeable effect that the path of the needle runs not radially, but in a circular arc. As far as the required perpendicular position of the needle plane is concerned, no substantial difficulties are encountered, provided that the attention is directed towards this issue. Thus, the task of achieving a tracking as correct as possible mainly comes down to the question of tangentiality of the needle plane. The angle between the needle plane and the groove tangent will be referred to here as the tracking error. The first step is to determine its magnitude.

From Fig. 1a and 1b, which doubtlessly don't require any further explanation, it is evident that the tracking error can be expressed as

$$\delta = \frac{\pi}{2} - (\varphi + \vartheta)\,, \tag{1}$$

where φ is the angle OPQ, which varies as a function of radius r during the playing process, and ϑ is the fixed angle QPS (the offset angle), to which the pick-up is adjusted in relation to the tone arm.

The shape of the curve $\varphi = f\,(r)$ is determined by the two lengths $QP = R$ and $OQ = D$, the effective arm length and the mounting distance, respectively, according to the equation

$$\cos\varphi = \frac{1}{2\,R\,r}\,(R^2 - D^2 + r^2)\,. \tag{2}$$

A descriptive picture of this relationship is given in Fig. 2, where curves of φ are drawn for 5 different values of D with a fixed R. The curve *IV* corresponds to the particular case that $D = R$, meaning that the needle tip passes exactly over the centre of the record, O. This curve *IV* is characterised by a rather large slope, and even more so curve *V*, where $D > R$. If on the other hand D is selected $< R$ (curves *I-III*), such that the needle tip follows the path PP', a maximum, φ_{max} is formed on the φ curve, which shifts to the right with decreasing D.

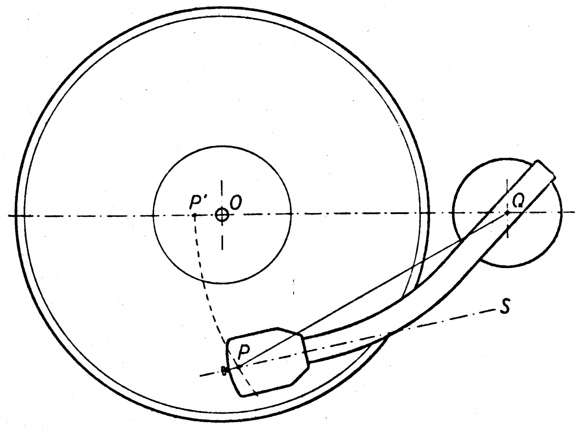

Fig. 1a. Tracking device with pivoted tone arm

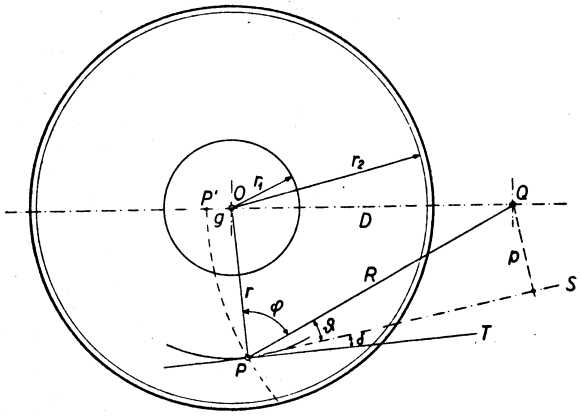

Fig. 1b. Introduction of the basic geometric parameters of the device of Fig. 1a

In the vicinity of this maximum, the curve is relatively flat, a circumstance which can be used to limit the tracking error in an appropriate way.

3. Conditions for smallest tracking error tolerance

In earlier publications it was merely required that the change in the tracking error should be as small as possible. This is only achieved when the two values of φ, φ_1 and φ_2 for radii r_1 and r_2 respectively (inner and outer groove radii) are set equal, and to then set the offset angle ϑ to the angle complementary to the arithmetic mean of

φ_1 and φ_{max}. The tracking error then lies between the limits ± ½ (φ_{max} − φ_1). The conditions for this configuration are according to [1-3]):

$$(3) \quad D = \sqrt{R^2 - r_1 r_2}\,,$$

$$(4) \quad \vartheta = \frac{1}{2}\left(\arcsin\frac{r_1 + r_2}{2R} + \arcsin\frac{\sqrt{r_1 r_2}}{R}\right)$$

$$(4a) \quad \simeq \arcsin\frac{r_1 + r_2 + 2\sqrt{r_1 r_2}}{4R}\,.$$

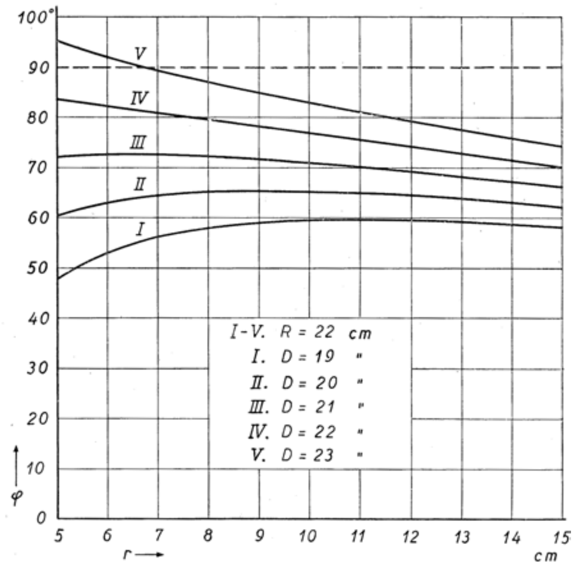

Fig.2 Variation of angle of the pickup during playback

On typical records of 30 cm diameter, the limits of the recorded radii are r_1= 5.0 cm and r_2 = 14.5 cm (see Appendix). The tone arm effective length R is mainly limited by space requirements, and in practice, is 20-25 cm in most cases.

In place of the mounting distance D and the offset angle ϑ, it is often useful to introduce another pair of parameters, namely the overhang $g = R - D$ (OP' in Fig. 1), which indicates how far the needle path runs outside the record centre, and the linear offset $p = R \sin\vartheta$ (see Fig. 1b), which is the distance from the needle plane to the tone arm pivot. Using the given dimensions, Equation (3) results in an overhang of 15-19 mm (dependent on R), and Equation (4a) results in a linear offset of 91 mm (independent of R).

For the maximum tracking error, for a configuration according to Equations (3) and (4a), one obtains the approximate formula

$$(5) \quad \delta_{max} \simeq \frac{r_1 + r_2 - 2\sqrt{r_1 r_2}}{4\sqrt{R^2 - \frac{1}{16}(r_1 + r_2 + 2\sqrt{r_1 r_2})^2}}\,.$$

For an effective arm length of $R \geq 20$ cm, Equation (5) gives a maximum tracking error $\delta_{max} \leq 2°$. In comparison, measurements performed by the author on gramophones and radio-gramophones found tracking errors of around 10° in many cases, and sometimes even more. With older players, tracking errors of even 20° - 30° are found [3]).

4. Non-linear distortion from non-tangential tracking

It could be assumed that the small angular error obtained according to the previous section would hardly be of any noticeable importance. Actually, far larger errors can occur in practice. Additionally, as will be demonstrated later, there are other criteria with respect to groove tracking that bring into consideration certain departures from the tangentiality goal. In order to understand the circumstances correctly, it is necessary to clarify the harmful influence of tracking error, and if possible, to express it quantitatively. The primary goal of this article is to study the theoretical nature and magnitude of the non-linear distortion. B. Olney addresses the same problem in a publication [8]) issued during the development of this work, but without indicating a general solution. He determines the distortion parameter in some individual cases by means of graphical harmonic analysis, but the results found are too small due to an error (see page 355).

It is easy to understand that non-tangential tracking of a sinusoidal groove cannot result in a purely sinusoidal deflection of the needle tip. Harmonics are generated, and the task now is to calculate these.

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[1-3]) ibidem

[8]) B. Olney, Phonograph pickup tracking error vs distortion and record wear. Electronics 10 (1937), no. 11, p. 19-23 and 81.

In fact, the grooves are not usually pure sine waves, but composed of a number of frequencies, whereby non-linearity in the playing of the record produces a range of intermodulation products (see section 5). Nevertheless, the distortion generated by a simple sinusoidal groove is of interest in that the distortion parameter can serve as a suitable relative measure of the non-linearities produced at various tracking errors. A certain value of the distortion parameter may however, as will be shown later, represent a distortion which is greater than that produced by e.g. tube amplifiers.

Fig. 3 shows a sine wave groove with amplitude A and wavelength λ.

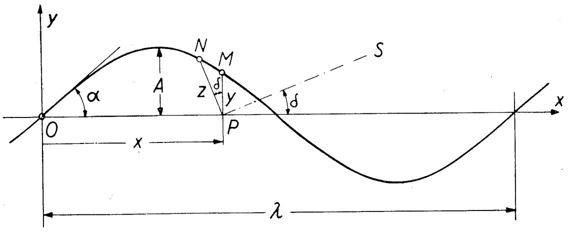

Fig.3 Determination of the distortion in the case of sinusoidal groove modulation

The groove may be considered a straight line because the groove amplitude is always very small in relation to the radius of curvature of the groove. For those frequencies for which non-linear tracking is of importance, we may further assume that the body of the pickup does not move in response to needle movements, but does move with a steady velocity with respect to the record. We will disregard the widths of the groove and the needle tip, and assume the curve of Fig. 3 represents the path of the needle tip. In fact, distortion is also generated as a result of deviation from this assumption, but this distortion, for which the means to reduce it have been suggested, represents a particular sub-problem which has been discussed in the literature [9,10]) and we will not complicate the current task by considering it here.

Assume that at a certain instant, P is the position of the needle tip without modulation of the groove, i.e. the rest position, M the corresponding needle tip position on the modulated groove with tangential tracking, and N the position on the same modulated groove with non-tangential tracking, with a tracking error δ. The moving coordinate of P is designated x, and the deflections PM and PN of the needle tip with tangential and non-tangential tracking are, respectively, y and z. Then y and z are defined by

$$y = A \sin \frac{2\pi x}{\lambda}, \tag{6}$$

$$z = \frac{A}{\cos\delta} \sin \frac{2\pi}{\lambda}(x - z \sin\delta). \tag{7}$$

Note that z is not an explicit function of the variable x. However, we want to calculate such an expression in the form of a Fourier Series, and thus write

$$z = \sum_{n=1}^{\infty} \left(A_n \sin \frac{2\pi n x}{\lambda} + B_n \cos \frac{2\pi n x}{\lambda} \right), \tag{8}$$

where

$$A_n = \frac{2}{\lambda} \int_0^{\lambda} z \sin \frac{2\pi n x}{\lambda} \, dx, \tag{9}$$

$$B_n = \frac{2}{\lambda} \int_0^{\lambda} z \cos \frac{2\pi n x}{\lambda} \, dx. \tag{10}$$

Since $z_{\lambda - x} = -z_x$, B_n must vanish for each value of n. For A_n, we find by partial integration

$$\begin{aligned} A_n &= -\frac{2}{\lambda} \left| z \frac{\lambda}{2\pi n} \cos \frac{2\pi n x}{\lambda} \right|_0^{\lambda} \\ &\quad + \frac{1}{\pi n} \int_0^{\lambda} \frac{dz}{dx} \cos \frac{2\pi n x}{\lambda} \, dx \\ &= \frac{1}{\pi n} \int_0^{\lambda} \frac{dz}{dx} \cos \frac{2\pi n x}{\lambda} \, dx. \end{aligned} \tag{11}$$

By using this form of the integral, it is easier to rid oneself of the implicit relationship between the two variables z and x. For this purpose, we introduce a new variable Θ defined by the equation

$$z = \frac{A}{\cos\delta} \sin\Theta. \tag{12}$$

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9) H.A. Frederick, Vertical sound records; recent fundamental advances in mechanical records on wax. J. Soc. Mot. Pict. Engrs. 18 (1932), no. 2, p. 141-163

10) J.A. Pierce and F.V. Hunt, On distortion in sound reproduction from phonograph records. J. Acoust. Soc. Am. 10 (1938), no. 1, p. 14-28

By comparison with Equation (7) one obtains

$$\Theta = \frac{2\pi}{\lambda}(x - z\sin\delta) = \frac{2\pi x}{\lambda} - \frac{2\pi A}{\lambda}\operatorname{tg}\delta\cdot\sin\Theta$$

or

$$(13)\qquad \frac{2\pi x}{\lambda} = \Theta + \varepsilon\sin\Theta\,,$$

where

$$(14)\qquad \varepsilon = \frac{2\pi A}{\lambda}\operatorname{tg}\delta\,.$$

This parameter ε, which has fundamental meaning for the distortion, can also be expressed as

$$(14a)\qquad \varepsilon = \operatorname{tg}\alpha\cdot\operatorname{tg}\delta$$

with α being the angle of the maximum slope of the groove curve (see Fig. 3).

With Θ as the variable, the integral (11) takes the following form:

$$A_n = \frac{A}{\pi n\cos\delta}\int_0^{2\pi}\cos\Theta\cos(n\Theta + n\varepsilon\sin\Theta)\,d\Theta$$

$$= \frac{A}{2\pi n\cos\delta}\left\{\int_0^{2\pi}\cos[(n-1)\Theta + n\varepsilon\sin\Theta]\,d\Theta + \int_0^{2\pi}\cos[(n+1)\Theta + n\varepsilon\sin\Theta]\,d\Theta\right\}$$

$$= \frac{A}{n\cos\delta}\{J_{n-1}(-n\varepsilon) + J_{n+1}(-n\varepsilon)\}$$

or more simply

$$(15)\quad A_n = (-1)^{n+1}\frac{A}{\cos\delta}\cdot\frac{2}{n\varepsilon}J_n(n\varepsilon)\,.$$

Here $J_n(n\varepsilon)$ is the Bessel Function of the first kind and nth order of argument $n\varepsilon$.

The sought Fourier series of z is thus

$$(16)\quad z = \frac{A}{\cos\delta}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{n\varepsilon}J_n(n\varepsilon)\sin\frac{2\pi n x}{\lambda}\,.$$

In order to obtain the needle deflection z as a function of time t, we need to replace $2\pi x/\lambda$ with ωt, where ω is the angular frequency. For quality of reproduction, however, it is not the deflection of the needle tip which is important, but the lateral velocity. Apart from the frequency response of the reproduction equipment (sound box – funnel horn and/or phonograph pick-up - amplifier - loudspeaker), the partial tones of the sound wave are present in the same ratios as they are in the time curve of the velocity. For a sinusoidal groove of maximum deflection A, if we designate the velocity with tangential tracking as V, then $V = \omega A$. During non-tangential tracking the instantaneous velocity is

$$(17)\quad v = \frac{dz}{dt} = \frac{V}{\cos\delta}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{\varepsilon}J_n(n\varepsilon)\cos n\omega t\,.$$

The amplitude ratios between the fundamental and the harmonics are thus

$$(18)\qquad J_1(\varepsilon) : J_2(2\varepsilon) : J_3(3\varepsilon)\quad \text{etc.}$$

The velocity of the nth partial tone is

$$(19)\qquad V_n = \frac{V}{\cos\delta}\cdot\frac{2}{\varepsilon}J_n(n\varepsilon)$$

or, using a development of the Bessel Function into a power series

$$(19a)\qquad V_n = \frac{V}{\cos\delta}\cdot\frac{(n\varepsilon)^{n-1}}{2^{n-1}(n-1)!}\left\{1 - \frac{\left(\frac{n\varepsilon}{2}\right)^2}{1!\,(n+1)} + \frac{\left(\frac{n\varepsilon}{2}\right)^4}{2!\,(n+1)(n+2)} - \cdots\right\}.$$

For the fundamental tone and the first two harmonics we thus obtain

$$V_1 = \frac{V}{\cos\delta}\cdot\left(1 - \frac{\varepsilon^2}{8} + \frac{\varepsilon^4}{192} - \cdots\right),$$

$$(19b)\quad V_2 = \frac{V}{\cos\delta}\cdot\varepsilon\left(1 - \frac{\varepsilon^2}{3} + \frac{\varepsilon^4}{24} - \cdots\right),$$

$$V_3 = \frac{V}{\cos\delta}\cdot\frac{9\varepsilon^2}{8}\left(1 - \frac{9\varepsilon^2}{16} + \frac{81\varepsilon^4}{640} - \cdots\right).$$

If the parameter ε is substantially smaller than 1, which it always is in practice, it appears that the second harmonic can be set as $V_2 \approxeq \varepsilon V_1$ with good approximation, and all higher harmonics can be neglected. This means that the sought distortion parameter can simply be set equal to ε.

In order to find which values ε and hence the distortion parameter can have, we must, according to Equation (14a), first estimate the maximum slope angle α of the groove. With Ω being the angular velocity of the record, the wavelength λ at angular frequency ω is

$$(20)\qquad \lambda = 2\pi r\frac{\Omega}{\omega}\,,$$

from which follows

$$(21)\qquad \operatorname{tg}\alpha = \frac{2\pi A}{\lambda} = \frac{\omega A}{\Omega r} = \frac{V}{\Omega r}\,.$$

The velocity probably seldom exceeds 10 cm/s [11]), which is the equivalent of a width of optical pattern of approximately 25 mm [12]). At the inner radius r_1 = 5 cm, one thus obtains at 78 revolutions per minute:

$$\mathrm{tg}\,\alpha = 10 \cdot 60 / (2\pi \cdot 78 \cdot 5) = 0.25,$$

i.e. a slope of the groove of 1 : 4. If, further, tg δ = 0.2, which corresponds to the often occurring tracking error δ = 11°, ε becomes ε = ¼ · 0.2 = 0.05 and the distortion parameter thus 5%. As is well known such distortion levels in amplifiers etc. will be associated with a significant degradation of the sound quality when reproducing some kinds of music, such as orchestral music [13,14]). The type of distortion being discussed in this article, as will be shown in the following, has a much more harmful effect on the quality of reproduction, for an identical distortion parameter.

When the tracking device is configured such that the tracking error δ at the inner grooves amounts to only 2° (tg δ = 0.035), ε is decreased to only ¼ · 0.035 ≅ 0.009, so the distortion parameter is less than 1%, a value which may always be satisfactory.

B. Olney [8]) does calculations on a worst-case maximum value of the ratio λ/A of 16.3 at 500 Hz. With a tracking error δ = 15° he finds the distortion parameter to be 4.1%. This is not correct according to the above calculations. With tg $\alpha = 2\pi / 16.3 = 0.385$ and tg δ = tg 15° = 0.268 one obtains ε = 0.103, a value which is still small enough to allow the distortion parameter to be set practically equal to ε, it is therefore 10.3%. This large difference seems to some extent to be based on the fact that Olney erroneously determined the distortion for the groove amplitude instead of the groove velocity. This alone causes results in a ratio of 1 : 2 of the distortion parameters.

On condition that the tracking error δ amounts to only a few degrees (δ < 10°), tg δ can be replaced with δ to a good approximation, which results in an expression of ε in a form that is better suited for the following explanations:

$$\varepsilon \cong \frac{V}{\Omega} \cdot \frac{\delta}{r}\,. \tag{22}$$

The first factor is independent of the respective position of the needle tip on the record, while the second factor changes continuously. At this point the attention is drawn to the fact that a larger tracking error δ is more acceptable at the outer grooves than at the inner grooves, because it is δ/r, not δ, which is decisive for the distortion parameter. If one takes a closer look on this issue, this is understandable without calculation. With decreasing radius, the groove modulations are compressed in the longitudinal direction, whereby tracking error entails ever larger distortion. This is why, as far as distortion is concerned, that for the calculation of the best configuration of a tone arm, the previous requirement for the limits in the tracking error δ to be as narrow as possible should be replaced by appropriate limits on the parameter δ/r. The influence this has will be shown in section 6.

5. Intermodulation products from multiple frequency components

In order to illustrate a more general case, we assume that the groove is modulated with several superposed frequencies of any number (N), and express the deflection of the needle tip for tangential and non-tangential tracking, in analogy with Equations (6) and (7) as follows:

$$y = f(x) = \sum_{i=1}^{N} A_i \sin\left(\frac{2\pi x}{\lambda_i} + \beta_i\right), \tag{23}$$

$$z = \frac{1}{\cos\delta} f(x - z\sin\delta)\,. \tag{24}$$

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[8]) ibidem

[11]) H.J. von Braunmühl und W. Weber, Einführung in die angewandte Akustik, Leipzig 1936, p. 106

[12]) G. Buchmann und E. Meyer, Eine neue optische Meßmethode für Grammophonplatten. Elektr. Nachr.-Techn. 7 (1930), no. 4, p. 147-152

[13]) W. Janovsky, Über die Hörbarkeit von Verzerrungen. Elektr. Nachr.-Techn. 6 (1929), no. 11, p. 421-439

[14]) H.J. von Braunmühl und W. Weber, Über die Störfähigkeit nichtlinearer Verzerrungen. Akust. Z.2 (1937), no. 3, p. 135-147

(The index i in the algebraic sum in Equation (23), also k and l later, has to be understood only as a subscript index; therefore the summation has nothing to do with a FOURIER Series.)

Since it was made clear in section 4 that in cases occurring in practice, one must consider mainly non-linear first order effects, we can now employ an approximation to determine the amplitudes of the generated harmonics and intermodulation products. If one sets $z \cos \delta = \check{z} = f(x - \check{z} \operatorname{tg} \delta)$, then it is easy to see that $\check{z}$ along with z can be developed into a power series after $\operatorname{tg} \delta$, which, with small tracking error δ and small values of $\operatorname{tg} \alpha_i = 2 \pi A_i / \lambda_i$, can be presupposed to be strongly convergent. With three terms this power series becomes

$$z = \frac{y}{\cos \delta}\left[1 - y' \operatorname{tg} \delta + \frac{1}{2}(y y'' + 2 y'^2) \operatorname{tg}^2 \delta - \cdots\right]. \tag{25}$$

According to (23) y' and y'' are

$$y' = 2\pi \sum_{i=1}^{N} \frac{A_i}{\lambda_i} \cos\left(\frac{2\pi x}{\lambda_i} + \beta_i\right),$$

$$y'' = -4\pi^2 \sum_{i=1}^{N} \frac{A_i}{\lambda_i^2} \sin\left(\frac{2\pi x}{\lambda_i} + \beta_i\right).$$

For the components occurring in (25) one then finds:

$$y y' = 2\pi \sum_{i=1}^{N} \sum_{k=1}^{N} \frac{A_i A_k}{\lambda_k} \sin\left(\frac{2\pi x}{\lambda_i} + \beta_i\right) \cdot \cos\left(\frac{2\pi x}{\lambda_k} + \beta_k\right),$$

$$y^2 y'' = -4\pi^2 \sum_{i=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} \frac{A_i A_k A_l}{\lambda_l^2} \sin\left(\frac{2\pi x}{\lambda_i} + \beta_i\right) \cdot \sin\left(\frac{2\pi x}{\lambda_k} + \beta_k\right) \cdot \sin\left(\frac{2\pi x}{\lambda_l} + \beta_l\right),$$

$$y y'^2 = 4\pi^2 \sum_{i=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} \frac{A_i A_k A_l}{\lambda_k \lambda_l} \sin\left(\frac{2\pi x}{\lambda_i} + \beta_i\right) \cdot \cos\left(\frac{2\pi x}{\lambda_k} + \beta_k\right) \cdot \cos\left(\frac{2\pi x}{\lambda_l} + \beta_l\right).$$

After the introduction of $x = \Omega r t$ and $\lambda_i = 2 \pi \Omega r / \omega_i$, we separate these products into simple sine components:

$$y y' = \frac{1}{2 \Omega r} \sum_{i=1}^{N} \sum_{k=1}^{N} \omega_k A_i A_k \{\sin[(\omega_i + \omega_k) t + \beta_i + \beta_k] + \sin[(\omega_i - \omega_k) t + \beta_i - \beta_k]\},$$

$$\begin{aligned} y^2 y'' = \frac{1}{4 \Omega^2 r^2} \sum_{i=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} \omega_l^2 A_i A_k A_l \{ & \sin[(\omega_i + \omega_k + \omega_l) t + \beta_i + \beta_k + \beta_l] \\ & - \sin[(\omega_i + \omega_k - \omega_l) t + \beta_i + \beta_k - \beta_l] - \sin[(\omega_i - \omega_k + \omega_l) t + \beta_i - \beta_k + \beta_l] \\ & + \sin[(\omega_i - \omega_k - \omega_l) t + \beta_i - \beta_k - \beta_l]\}, \end{aligned}$$

$$\begin{aligned} y y'^2 = \frac{1}{4 \Omega^2 r^2} \sum_{i=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} \omega_k \omega_l A_i A_k A_l \{ & \sin[(\omega_i + \omega_k + \omega_l) t + \beta_i + \beta_k + \beta_l] \\ & + \sin[(\omega_i + \omega_k - \omega_l) t + \beta_i + \beta_k - \beta_l] + \sin[(\omega_i - \omega_k + \omega_l) t + \beta_i - \beta_k + \beta_l] \\ & + \sin[(\omega_i - \omega_k - \omega_l) t + \beta_i - \beta_k - \beta_l]\}. \end{aligned}$$

From this we obtain the deflection

$$\begin{aligned} z = \frac{1}{\cos \delta} \Bigg[& \sum_{i=1}^{N} A_i \sin(\omega_i t + \beta_i) - \frac{\operatorname{tg} \delta}{2 \Omega r} \sum_{i=1}^{N} \sum_{k=1}^{N} \omega_k A_i A_k \{\sin[(\omega_i + \omega_k) t + \beta_i + \beta_k] \\ & + \sin[(\omega_i - \omega_k) t + \beta_i - \beta_k]\} \\ & + \frac{\operatorname{tg}^2 \delta}{8 \Omega^2 r^2} \sum_{i=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} \omega_l A_i A_k A_l \{(2 \omega_k + \omega_l) \sin[(\omega_i + \omega_k + \omega_l) t + \beta_i + \beta_k + \beta_l] \\ & + (2 \omega_k - \omega_l) \sin[(\omega_i + \omega_k - \omega_l) t + \beta_i + \beta_k - \beta_l] + (2 \omega_k - \omega_l) \sin[(\omega_i - \omega_k + \omega_l) t \\ & + \beta_i - \beta_k + \beta_l] + (2 \omega_k + \omega_l) \sin[(\omega_i - \omega_k - \omega_l) t + \beta_i - \beta_k - \beta_l]\} \cdots \Bigg]. \end{aligned} \tag{26}$$

Finally, after introducing $\omega_i A_i = V_i$, the instantaneous velocity becomes:

$$
\begin{aligned}
v = \frac{dz}{dt} = \frac{1}{\cos\delta}\Bigg[& \sum_{i=1}^{N} V_i \cos(\omega_i t + \beta_i) - \frac{\operatorname{tg}\delta}{2\Omega r} \sum_{i=1}^{N}\sum_{k=1}^{N} V_i V_k \Big\{ \frac{\omega_i + \omega_k}{\omega_i} \cos[(\omega_i + \omega_k) t + \beta_i + \beta_k] \\
& + \frac{\omega_i - \omega_k}{\omega_i} \cos[(\omega_i - \omega_k) t + \beta_i - \beta_k] \Big\} \\
& + \frac{\operatorname{tg}^2\delta}{8\Omega^2 r^2} \sum_{i=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} V_i V_k V_l \Big\{ \frac{(2\omega_k + \omega_l)(\omega_i + \omega_k + \omega_l)}{\omega_i \omega_k} \cos[(\omega_i + \omega_k + \omega_l) t + \beta_i + \beta_k + \beta_l] \\
& + \frac{(2\omega_k - \omega_l)(\omega_i + \omega_k - \omega_l)}{\omega_i \omega_k} \cos[(\omega_i + \omega_k - \omega_l) t + \beta_i + \beta_k - \beta_l] \\
& + \frac{(2\omega_k - \omega_l)(\omega_i - \omega_k + \omega_l)}{\omega_i \omega_k} \cos[(\omega_i - \omega_k + \omega_l) t + \beta_i - \beta_k + \beta_l] \\
& + \frac{(2\omega_k + \omega_l)(\omega_i - \omega_k - \omega_l)}{\omega_i \omega_k} \cos[(\omega_i - \omega_k - \omega_l) t + \beta_i - \beta_k - \beta_l] \Big\} \cdots \Bigg].
\end{aligned}
\tag{27}
$$

When considering the permutations possible, we obtain the following velocities for various frequencies, without considering the common factor $1/\cos\,\delta$ and also without considering higher-order terms:

(28)

Angular frequency	Amplitude
ω_i	V_i
$2\omega_i$	$\frac{\operatorname{tg}\delta}{\Omega r} \cdot V_i^2$
$\omega_i + \omega_k$	$\frac{\operatorname{tg}\delta}{2\Omega r} \cdot \frac{(\omega_i + \omega_k)^2}{\omega_i \omega_k} V_i V_k$
$\lvert\omega_i - \omega_k\rvert$	$\frac{\operatorname{tg}\delta}{2\Omega r} \cdot \frac{\lvert\omega_i - \omega_k\rvert^2}{\omega_i \omega_k} V_i V_k$
$3\omega_i$	$\frac{9\operatorname{tg}^2\delta}{8\Omega^2 r^2} \cdot V_i^3$
$2\omega_i + \omega_k$	$\frac{\operatorname{tg}^2\delta}{8\Omega^2 r^2} \cdot \frac{(2\omega_i + \omega_k)^3}{\omega_i^2 \omega_k} V_i^2 V_k$
$\lvert 2\omega_i - \omega_k\rvert$	$\frac{\operatorname{tg}^2\delta}{8\Omega^2 r^2} \cdot \frac{\lvert 2\omega_i - \omega_k\rvert^3}{\omega_i^2 \omega_k} V_i^2 V_k$
$\omega_i + \omega_k + \omega_l$	$\frac{\operatorname{tg}^2\delta}{4\Omega^2 r^2} \cdot \frac{(\omega_i + \omega_k + \omega_l)^3}{\omega_i \omega_k \omega_l} V_i V_k V_l$
$\lvert\omega_i + \omega_k - \omega_l\rvert$	$\frac{\operatorname{tg}^2\delta}{4\Omega^2 r^2} \cdot \frac{\lvert\omega_i + \omega_k - \omega_l\rvert^3}{\omega_i \omega_k \omega_l} V_i V_k V_l$
$\lvert\omega_i - \omega_k - \omega_l\rvert$	$\frac{\operatorname{tg}^2\delta}{4\Omega^2 r^2} \cdot \frac{\lvert\omega_i - \omega_k - \omega_l\rvert^3}{\omega_i \omega_k \omega_l} V_i V_k V_l$.

A comparison with the basic case of non-linear distortion which may occur with tube amplifiers, for example, is of great interest here. The secondary, distorted parameter v is related to the primary, undistorted parameter u by the non-linear relationship

$$v = c_0 + c_1 u + c_2 u^2 + c_3 u^3 + \cdots \tag{29}$$

wherein

$$u = \sum_{i=1}^{N} U_i \sin(\omega_i t + \beta_i)\,. \tag{30}$$

The amplitudes of the various frequency components of v can be calculated in a way similar to the above. After the introduction of $c_1 U_i = V_i$ the set of these amplitudes looks as follows:

(31)

Angular frequency	Amplitude
ω_i	V_i
$2\omega_i$	$\frac{c_2}{2c_1^2} V_i^2$
$\omega_i + \omega_k$	$\frac{c_2}{c_1^2} V_i V_k$
$\lvert\omega_i - \omega_k\rvert$	$\frac{c_2}{c_1^2} V_i V_k$
$3\omega_i$	$\frac{c_3}{4c_1^3} V_i^3$
$2\omega_i + \omega_k$	$\frac{3c_3}{4c_1^3} V_i^2 V_k$
$\lvert 2\omega_i - \omega_k\rvert$	$\frac{3c_3}{4c_1^3} V_i^2 V_k$
$\omega_i + \omega_k + \omega_l$	$\frac{3c_3}{2c_1^3} V_i V_k V_l$
$\lvert\omega_i + \omega_k - \omega_l\rvert$	$\frac{3c_3}{2c_1^3} V_i V_k V_l$
$\lvert\omega_i - \omega_k - \omega_l\rvert$	$\frac{3c_3}{2c_1^3} V_i V_k V_l$.

Here all the components are independent from the frequency ratios. In (28), however, for the intermodulation products a factor comes into play which depends on the ratios of the interacting frequencies in such a manner that high frequency intermodulation products can take on rather large values.

This shall be demonstrated with the aid of a numerical example. We assume that only the harmonics and the second order intermodulation products are noticeable, and set for instance $\omega_2 = 8\,\omega_1$, and $V_2 = ¼\,V_1$, which could easily occur, for example, at $\omega_2 / 2\pi \triangleq 300$ Hz [15]). Allow the distortion parameter of the V_1 component to be 3%. With V_1 as the reference level, we then obtain the following amplitudes of the different frequency components in the two cases, according to (28) and (31).

Angular frequency	Amplitude	
	Eqn (28)	Eqn (31)
ω_1	1.000	1.000
$\omega_2 = 8\,\omega_1$	0.250	0.250
$2\,\omega_1$	0.030	0.030
$2\,\omega_2 = 16\,\omega_1$	0.002	0.002
$\omega_1 + \omega_2 = 9\,\omega_1$	0.038	0.015
$\omega_2 - \omega_1 = 7\,\omega_1$	0.023	0.015

The disturbance effect is predominately determined by the amplitudes of the last two components. The pure octave components cause little disturbance. In the above example, considerably larger distortion of the sound would result with (28) than with (31), although the distortion parameters are the same in both cases. It is also enlightening that in the case of (28), neither the summation tone alone nor the difference tone alone can serve as a useful distortion measure for a comparison with the simple case of distortion according to (31). More suitable would be the RMS value of both, which amounts to 0.044 according to (28), and 0.021 according to (31) (which is equivalent to 27 and 34 dB, respectively, below the levels of the original tones. This RMS value also depends, in the previous case (28), on the frequency ratio, and rises approximately proportionally to that ratio if one of the two frequencies becomes multiple times larger than the other one.

From these considerations, it follows that the non-linear distortion which may be generated during the reproduction of records because of tracking error, exhibits a different character from, say, the distortion from a tube amplifier at high gain. The risk of distortion of non-tangential tracking error lies in the generation of higher frequency intermodulation products which can cause a dirtiness in the reproduction of the higher frequency ranges. It has also been made clear that here an importance different from the usual must be attached to the distortion parameter. However, the distortion parameter is well suited as a relative measure of the distortion effects resulting from different tracking errors. The value of the distortion factor itself that is acceptable for good reproduction quality depends strongly on the material in the recording. Further, it is the most difficult case (which is probably orchestral music) which determines the requirements for the tracking device. Consequently, it is appropriate to aim for a distortion parameter that is substantially smaller in value than what one would normally consider to be permissible.

In the opinion of the author, the usefulness of the distortion parameter as a comparative measure of non-linear distortion is mostly overrated. One should always deal carefully with it and, unless it is not a simple comparison between distortions of one and the same type, one should possibly perform an investigation of the generated intermodulation products similar to the above.

6. Conditions for the smallest distortion parameter tolerance

The task of finding the most favourable design (see Section 3) will now be modified in that the linear offset p and the overhang g which achieve the lowest tolerance of the distortion parameter and hence the parameter δ/r is to be determined.

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[15]) L.J. Sivian, H.K. Dunn and S.D. White, Absolute amplitudes and spectra of certain musical instruments and orchestras. J. Acoust. Soc. Am. 2 (1931), no. 3, p. 330-371

Once again, we proceed from the fundamental Equation (2), in which the following approximation is permissible:

$\cos\varphi = \cos(\varphi_0 - \delta) \cong \cos\varphi_0 + \delta \sin\varphi_0$ or

$$\delta \cong \frac{\cos\varphi - \cos\varphi_0}{\sin\varphi_0}, \tag{32}$$

where φ_0 represents the angle φ with tangential tracking.

For the parameter δ/r in which we are interested, we therefore obtain

$$\frac{\delta}{r} \cong \frac{1}{r\sin\varphi_0}\left(\frac{R^2 - D^2 + r^2}{2Rr} - \cos\varphi_0\right). \tag{33}$$

In order that this expression becomes equal for $r = r_1$ and $r = r_2$, it is necessary that

$$\cos\varphi_0 = \frac{R^2 - D^2}{2R}\left(\frac{1}{r_1} + \frac{1}{r_2}\right), \tag{34}$$

where

$$\frac{\delta_1}{r_1} = \frac{\delta_2}{r_2} = \frac{r_1 r_2 - R^2 + D^2}{2Rr_1 r_2 \sin\varphi_0}. \tag{35}$$

(As the factor sin φ_0 drops out later, it can therefore remain constant.) δ/r reaches its maximum at the radius

$$r^* = \frac{R^2 - D^2}{R\cos\varphi_0}, \tag{36}$$

or, if at the same time $\delta_1/r_1 = \delta_2/r_2$, then

$$r^* = \frac{2r_1 r_2}{r_1 + r_2}. \tag{37}$$

Then:

$$\left(\frac{\delta}{r}\right)_{max} = -\frac{1}{2R\sin\varphi_0}\left[\frac{R^2 - D^2}{4}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)^2 - 1\right]. \tag{38}$$

If one now sets $\delta_1/r_1 = -(\delta/r)_{max}$, one obtains

$$D = \sqrt{R^2 - a^2}, \tag{39}$$

where

$$a^2 = \frac{8(r_1 r_2)^2}{4r_1 r_2 + (r_1 + r_2)^2}, \tag{40}$$

and further

$$\cos\varphi_0 = \frac{p}{R}, \tag{41}$$

where

$$p = \frac{a^2}{r^*} = \frac{4r_1 r_2(r_1 + r_2)}{4r_1 r_2 + (r_1 + r_2)^2}. \tag{42}$$

Also in this case the linear offset p is independent of the effective arm length R.

For the maximum value of the parameter δ/r, one obtains finally

$$\left|\frac{\delta}{r}\right|_{max} = \frac{\frac{p}{r^*} - 1}{2\sqrt{R^2 - p^2}}. \tag{43}$$

With the earlier used recorded radii of $r_1 = 5.0$ cm and $r_2 = 14.5$ cm, the linear offset p becomes 8.5 cm, hence somewhat smaller than for the design for the smallest tracking error tolerance. Furthermore $a^2 = 62.8$, from which the mounting distance D and the overhang g for various effective arm lengths R are calculated (dimensions in cm):

R	20	22	23.5	25	27
D	18.4	20.5	22.1	23.7	25.8
G	1.6	1.5	1.4	1.3	1.2

The maximum distortion parameter, calculated according to (22) and (43) and based on the earlier values of 10 cm/s velocity and 78 revolutions per minute, is less than 0.5% for this optimal configuration.

An objection that could be raised against the above calculations is that the three maximum values of the parameter δ/r are not of the same importance. A greater importance should actually be attached to the maximum at r^* than to the maxima at the inner and outer recorded radii r_1 and r_2, first because δ/r changes only slowly in the vicinity of r^*, while in contrast δ/r changes very rapidly at r_1 and r_2. Secondly, the inner and outer radii r_1 and r_2 are not necessarily utilised with each record. Because of this consideration one should permit somewhat larger values of δ/r at r_1 and r_2 than at r^*, which can be obtained by a small reduction in the offset angle (see Fig. 2).

The greater importance of the maximum at r^* can be taken into account through the use of the method of least squares. Here the integral

$$\int_{r_1}^{r_2} \left(\frac{\delta}{r}\right)^2 dr$$

has to be minimised when varying D and φ_0 (or g and p, respectively).

In accordance with this method, we will introduce an effective distortion parameter. It shows that the design for its smallest value departs only slightly from the design indicated above. This design is, however, much more laborious.

7. Discussion of configurations other than the optimum

Even if the design of a playback apparatus is based on the design rules indicated above, because of the variance in manufacture one must always expect that alignment errors other than the theoretically calculated tracking error still occur, and which can hardly be considered as negligibly small. This is one reason why it is of interest to know how the distortion changes when the configuration differs more or less from the optimum. Furthermore, there arises in practice the case where one wants to use a particular pick-up with a non-optimum angular offset, while having flexibility concerning the mounting of the pick-up relative to the platter. The question arises, which arrangement is the most favourable with regard to distortion.

For the evaluation of different configurations with regard to distortion, the maximum distortion parameters are not very suitable because for each case, it would be necessary to indicate three numbers: the two end maxima and the mid-point maximum. Since these maxima, as previously mentioned, are not of equal importance, it is not sufficient to indicate only the largest of them. A more suitable quality measure is provided by the effective distortion parameter, defined by

$$K_{eff} = \sqrt{\frac{1}{r_2 - r_1} \int_{r_1}^{r_2} K^2 \, dr}\,, \tag{44}$$

with K being the familiar distortion parameter. In order to obtain an illustrative picture of how this parameter K_{eff} is dependent of the configuration, the author undertook numerical calculations for an average arm length, R = 22 cm, and different combinations of linear offset p and overhang g. The distortion parameter was calculated according to the approximate formula $K \cong V/\Omega \cdot \delta/r$, with the assumption that V = 10 cm/s and $\Omega = 2\pi \cdot 78/60$ radians per second; δ was calculated according to Equations (1) and (2). Using the calculated values, curves of equal effective distortion parameter K_{eff} are drawn in a diagram with the axes being p and g, in Fig. 4. The curves *I-IV* correspond to K_{eff} values of 0.5, 1, 1.5 and 2%.

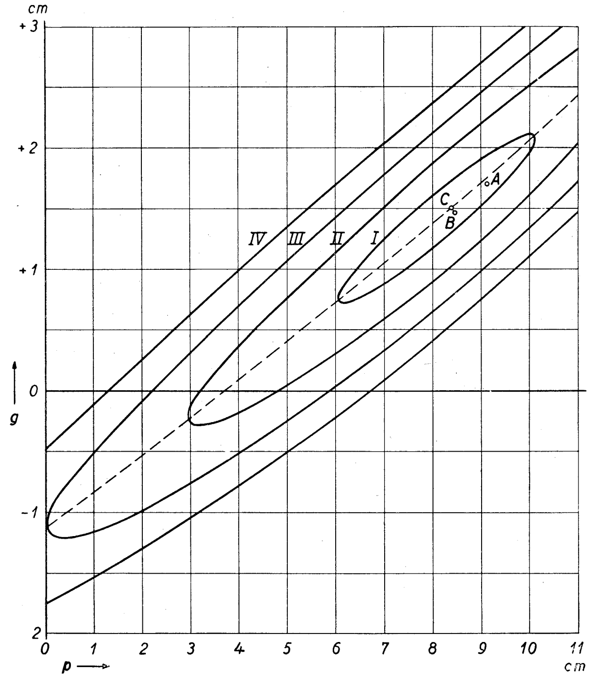

Fig.4 Curves of equal effective distortion parameter

The points A, B and C in the p-g plane represent different optimal configurations, where A has the smallest tracking error tolerance, B the smallest distortion parameter tolerance, and C the smallest effective distortion parameter. In the latter, K_{eff} amounts to 0.25%.

From the shape and location of the curves in Fig. 4 it results that the largest distortion risk occurs when the overhang is not correctly set for the linear offset. On the other hand, the angular offset itself is not so critical. How the overhang is to be selected for a given linear offset is shown by the dashed curve, the equation for which will be derived further down.

For an arbitrarily selected linear offset p, we want to determine that value of overhang g for which the effective distortion parameter K_{eff} is a minimum. Provided that the tracking error δ is relatively small, then from (22), (33), (39) and (41):

$$(45)\quad K \cong |\varepsilon| \cong \frac{V}{\Omega} \cdot \left|\frac{\delta}{r}\right| = \frac{V}{\Omega} \cdot \frac{1}{\sqrt{R^2 - p^2}} \cdot \left|\frac{1}{2} - \frac{p}{r} + \frac{a^2}{2r^2}\right|$$

And thus:

$$K_{eff}^2 = \frac{1}{r_2 - r_1} \int_{r_1}^{r_2} K^2 \, dr$$

$$(46)\quad \cong \frac{V^2}{\Omega^2} \cdot \frac{1}{(r_2 - r_1)(R^2 - p^2)} \left[\frac{r_2 - r_1}{4} - p \ln \frac{r_2}{r_1} + \left(p^2 + \frac{a^2}{2}\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - \frac{p\,a^2}{2}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) + \frac{a^4}{12}\left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right)\right].$$

A change in g is equivalent to a change in a^2, since

$$(47)\quad g = R - \sqrt{R^2 - a^2}$$

$$(47a)\quad \cong \frac{a^2}{2R}.$$

We therefore set $\frac{d\,(K^2_{eff})}{d\,(a^2)} = 0$ and obtain

$$(48)\quad a^2 = \frac{3\,r_1 r_2 \,[p\,(r_1 + r_2) - r_1 r_2]}{r_1^2 + r_1 r_2 + r_2^2}.$$

In (47, 48), we have the sought relationship between linear offset and overhang. If we replace (47) with (47a), we obtain the solution, as a good approximation, the straight line,

$$(49)\quad g \cong \frac{3\,r_1 r_2 \,[p\,(r_1 + r_2) - r_1 r_2]}{2\,R\,(r_1^2 + r_1 r_2 + r_2^2)},$$

Or with the previous values of r_1 and r_2,

$$(49a)\quad g \cong 6{,}89 \cdot \frac{p - 3{,}72}{R}$$

(dimensions in cm)

With a linear offset of 37 mm, the overhang will be zero, i.e., the path of the needle tip passes through the record centre, indeed independently of the arm length. A smaller linear offset, which is, however, not recommended, requires a negative overhang. With a tone arm of the older type, without any angular offset of the pick-up, the needle tip would pass about 12 mm inside the record centre, depending on the arm length.

In finishing, we want to briefly mention another aspect which is associated with this manner of tracking, namely record wear. P. WILSON and G.W. WEBB [2]) have pointed out that a variation in the tracking error is harmful in this regard, because the sharpened edge of the needle acts as a cutting tool on the groove walls. From Fig. 2 it is easily understood that when the φ curve runs as horizontally as possible, the angle φ changes the least, i.e., the change in tracking error is the least. Based on this criterion, a design according to section 3 would be the most favourable (point A in Fig. 4). In addition, there is another type of wear which is based on the fact that generally, a one-sided pressure component is present between the needle tip and the groove wall, which is proportional to cos φ. In order to keep this lateral pressure small, we would have to work with a curve in Fig. 2 which is positioned higher up, such as curve V or even higher, which would require a small, possibly even negative, angular offset. Thus, these two wear phenomena place completely different demands on the configurations of the tracking device. It seems then that a compromise solution was unable to be found which provides for the longest life expectancy of records. However, the experiences of the author indicate that the design for smallest distortion can also adequately satisfy the requirements for least record wear. This design actually forms a certain compromise between the smallest tracking error variation and the smallest lateral pressure.

Annex
Inner and outer recorded radii for standard records

In order to obtain a reliable basis for numerical calculations, the author has had measurements made of the inner and outer recorded radii (r_1 and r_2) of a large number of standard records.

[2]) ibidem

More than 1000 25-cm records were measured and almost as many 30-cm records, representing 10 different record manufacturers.

Since standard records (as against motion picture records) are recorded outside to inside, the outer recorded radius is within very close limits, whereas the inner radius, as is easily understandable, shows large variance. For 25-cm records one can expect an outer radius r_2 of 12.0 cm and for 30-cm records 14.5 cm. For the inner radius the curves of frequency of distribution as shown in Fig. 5 were obtained.

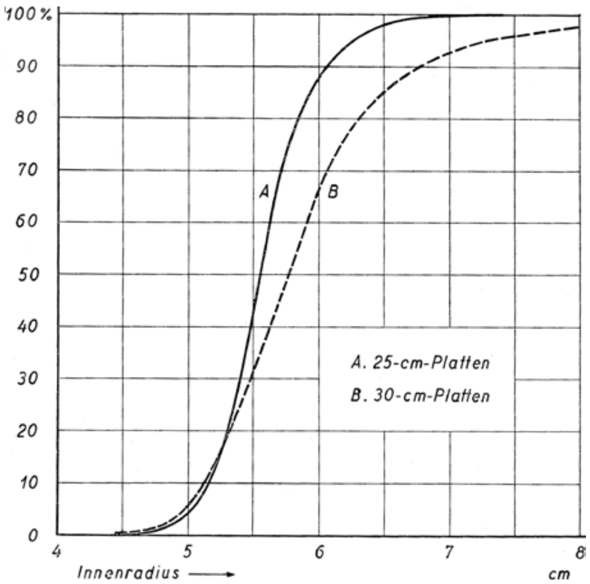

Fig. 5 Frequency of distribution of different inner radii on standard records

The ordinate indicates what percentage of the measured records had an inner radius smaller than the value given on the abscissa. Curve A refers to 25-cm record, curve B to 30-cm records.

For practical reasons the measurements were performed at the outermost, unmodulated grooves. If one considers that the modulation stops about 2 mm before the outlet groove, one must increase the values of the abscissa of the curves by this amount, in order to arrive at the correct inner radius r_1. From the curves, it is then evident that in order to be on the safe side in the vast majority of cases, we can use a minimum inner radius of 5.0 cm. Only in one or two cases per hundred does it occur that the inner radius is less than this value, and then it is usually only just slightly less. It is important for the design of the tracking device that the inner radius r_1 is not chosen smaller than necessary, because otherwise tracking error and distortion have to be kept somewhat higher, and this is only for the rare, exceptional cases.

Abstract

The generally used tracking device for records using a pivoted tone arm which holds the pickup shows an imperfection that has been recognised since long, and consists of the fact that the longitudinal axis of the pickup needle does not in general coincide exactly with the tangential direction of the groove. This can be the case in at most two points along the needle path, while everywhere else a certain tracking error is present, which varies during playback. The present work aims at the theoretical examination of the type and amount of the non-linear distortion generated because of this tracking error. By mathematical analysis of the simple case that the groove is modulated by a pure sine wave, first the distortion parameter is determined. Furthermore the resulting intermodulation products generated are calculated when several sine wave components are superposed. It becomes apparent that the distortion is of a completely different nature as compared to the basic non-linear distortion from, for example, a tube amplifier. The intermodulation products strongly depend on the frequency ratios, and can take relatively high values. Therefore, for a given distortion parameter, the resulting distortion can be substantially greater than that produced by a tube amplifier for the same distortion parameter. With regard to distortion the optimum configuration of the tracking device is determined; it differs somewhat from the configuration for smallest tracking error tolerance. Finally, configurations other than the optimum are discussed and evaluated with the help of an effective distortion parameter.

(Received on 26 July 1938.)

Translator's Notes

The original paper on which this translation is based appeared in November 1938 in Akustische Zeitschrift (Germany), Vol.3, pps.350-362, with the title "Über die nichtlineare Verzerrung bei der Wiedergabe von Schallplatten infolge Winkelabweichungen des Abtastorgans".

Permission to put the original paper in the public domain was obtained from the editor, S. Hirzel Verlag GmbH & Co., Stuttgart, Germany, on 23 November 2000. The original paper is available at http://www.helices.org/auDio/turnTable/

Involved in the reviewing process of the translation were lemko, Jas and Mark Kelly from the Vinyl Engine (www.vinylengine.com), as well as Graeme Dennes, who further provided valuable comments and who took care of the final layout. Thanks to all who participated in this project.

In 1924 Percy Wilson published his pioneering analysis on minimising record wear and tracking distortion by minimising the angular tracking error, and developed offset angle and overhang equations to achieve this. In 1938, Erik Löfgren was the first to provide an analytical treatment of tracking distortion, and developed new equations for offset angle and overhang which would minimise tracking distortion by minimising the weighted tracking error. Later, works by Baerwald (1941), Bauer (1945), Seagrave (1956) and Stevenson (1966) followed, all of whom presented design equations for optimum offset angle and overhang which are mathematically identical to those of Löfgren, as shown in an analysis by Graeme Dennes in 1983. In a letter to the editor, published in the May 1983 issue of Audio, Graeme states: "However, it should be stated clearly that if Löfgren's paper had been the only paper ever published on the subject, we would still have the same optimum design equations to employ in the reduction of tracking distortion as we have today."

The Hague, Netherlands, November 2008
Klaus Rampelmann